NAME: _____ **Teacher (circle): EB/JO/LE/VI/WH**

BERGVLIET HIGH SCHOOL

MATHEMATICS DEPARTMENT

GRADE 11

MATHEMATICS PAPER 2

NOVEMBER EXAMINATION 2024

5 November 2024

MARKS: **125**

TIME: **2 hour 30 minutes**

SET BY: **S. WHITEHORN**

MODERATOR: B. JORGE

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of **10** questions and **19** pages.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round answers off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet, with formulae, is included on the **last page** of the question paper.
9. Write legibly and present your work neatly.

Q 1	Q 2	Q 3	Q 4	Q 5	Q 6	Q 7	Q 8	Q 9	Q10	TOTAL
15	16	12	27	10	6	16	13	4	6	125

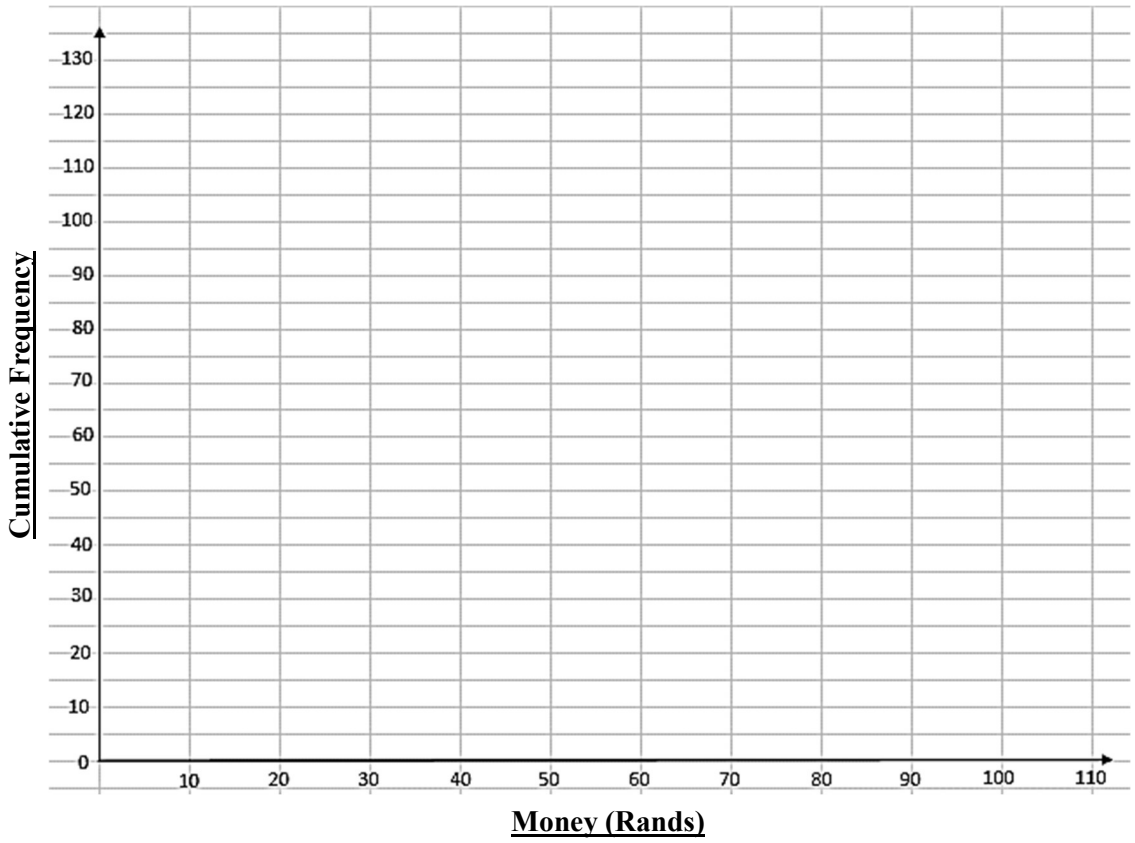
QUESTION 1

In a survey conducted at BHS, the amount of money spent at the tuckshop by learners in a week was recorded. The results are shown in the frequency table below:

Total spent x (R)	Frequency	Cumulative Frequency
$40 < x \leq 50$	5	
$50 < x \leq 60$	11	
$60 < x \leq 70$		32
$70 < x \leq 80$	21	
$80 < x \leq 90$		93
$90 < x \leq 100$	27	

1.1. Complete the missing values in the frequency and cumulative frequency columns in the table above. (2)

1.2. Use the axes provided below to draw an ogive for the given data set. (3)



1.3. Estimate the number of learners who spend more than R75 at the tuckshop. (2)

1.4. The prefects at BHS wanted to determine the amount of time (in minutes) students spent in line at the tuckshop. They want to try and decrease the waiting time so that the tuckshop is not overwhelmed by students.

The data they collected on their first day is shown below:

13	15	9	12	12
8	5	2	6	7

1.4.1. Determine the mean time students waited in line. (1)

1.4.2. Determine the standard deviation for the data set. (2)

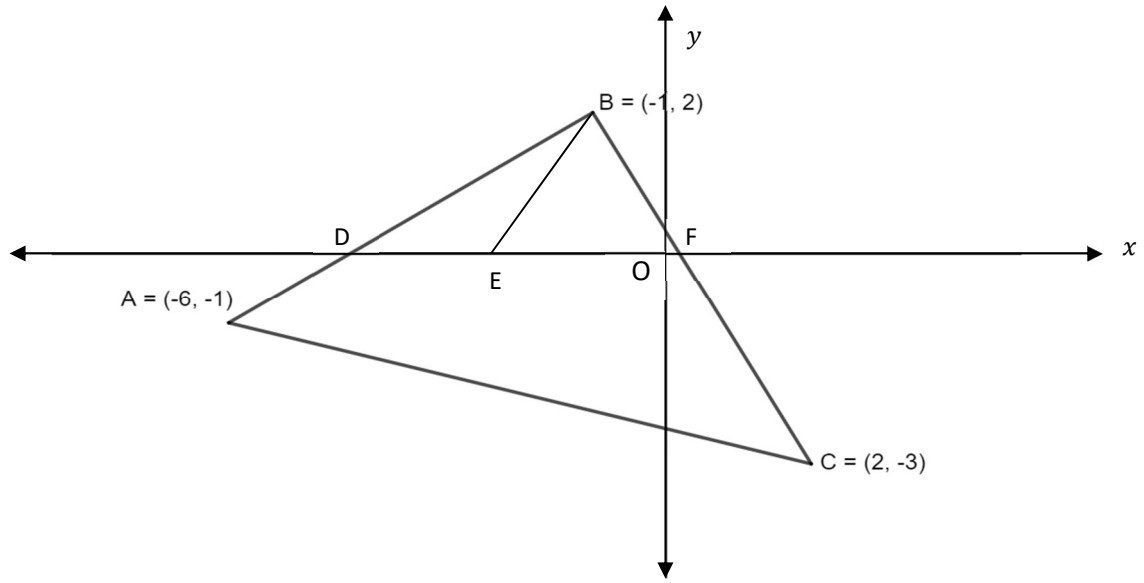
1.4.3. The prefects decided that waiting longer than one standard deviation from the mean was too long. How many students waited too long in the tuckshop line? (2)

1.4.4. Two more students informed the prefects that they both waited in line at the tuckshop for x minutes. When these two times are now considered, the new mean waiting time is $9{,}25$ minutes. Determine how long the two students waited in line. (3)

[15]

QUESTION 2

In the diagram below, ΔABC is given with $A(-6;-1), B(-1\,;2)$ and $C(2;-3)$. D and F are points on the x-axis of AB and BC respectively. E is a point on the x-axis.



2.1. Determine the equation of BC. (3)

2.2. Determine the coordinate of M, the fourth vertex of parallelogram $ABCM$ if M is in quadrant III. (2)

2.3. Prove that AC is the diameter of the circle passing through A, B and C, with reasons. (2)

2.4. Determine the size of $B\hat{A}C$. (3)

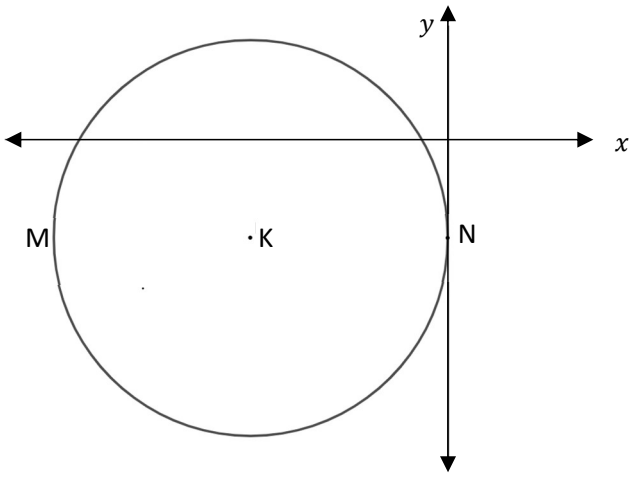
2.5. Write down the equation of the circle passing through A, B and C. (3)

2.6. Given that $F\left(\frac{1}{5}; 0\right), DF = \frac{68}{15}$ units and $DE: EF = 3: 5$, determine the coordinates of E. (3)

[16]

QUESTION 3

In the diagram below, circle centred at K has the equation $x^2 + y^2 + 8x + 4y = -4$. Circle K touches the y −axis at N.



3.1. Determine the centre coordinates and radius of the circle. (4)

3.2. Determine the coordinates of M, a point on the circumference of diameter MN. (2)

3.3. The circle centred at K is translated 3 units up, then reflected about the x-axis, to form a new circle with centre P. Write down the equation of this new circle in the form $(x - a)^2 + (y - b)^2 = r^2$. (2)

3.4. The radius of circle centre P is increased by 1 unit. Show that this new, enlarged circle, will intersect with the circle centred at K. Support your answer with appropriate calculations. (4)

[12]

QUESTION 4

4.1. If $\sin 34° = p$, determine the following in terms of p:

Sketch:

4.1.1. $\cos 56°$ (1)

4.1.2. $\sin 214°$ (2)

4.1.3. $\tan(-34°)$ (3)

4.2. Simplify the following without the use of a calculator: (7)

$$\frac{\tan(210°).\sin(480°).\sin(10°)}{\cos(-80°).\cos(225°).\sin(315°)}$$

4.3. Determine the general solution of: $\sin^2\theta - \sin\theta\cos\theta = 0$. (5)

4.4. Given the identity: $\cos\theta\,(1 + \tan^2\theta) = \frac{1}{\cos\theta}$

4.4.1. Prove the above identity. (3)

4.4.2. For which values of θ is the above identity undefined? (2)

4.5. Simplify the following without the use of a calculator: (4)

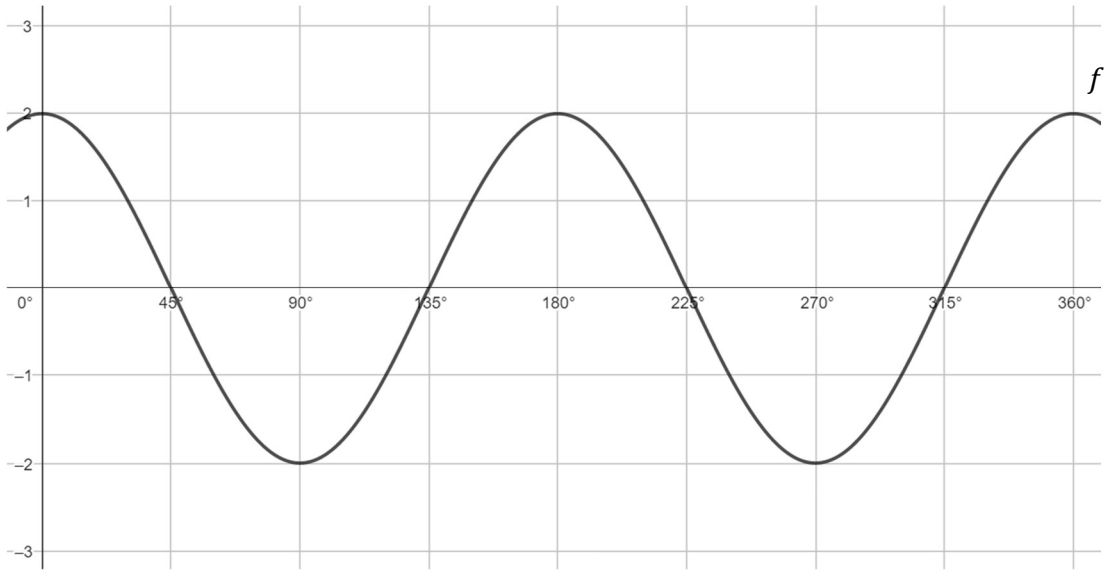
$$\sqrt{4^{\sin 150^\circ} . 2^{3 \tan 225^\circ}}$$

[27]

EXTRA SPACE

QUESTION 5

Given: $f(x) = a\cos bx$ for $x \in [0°; 360°]$.



5.1. Determine the values of a and b. (2)

5.2. Write down the period of f. (1)

5.3. Determine the values of x for which $f(x) < 0$ for $x \in [0°; 180°]$. (2)

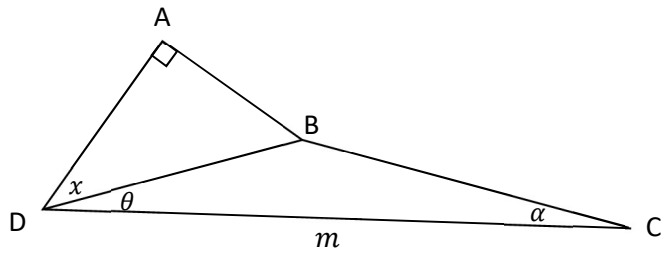
5.4. On the same axes as f above, sketch the graph of $g(x) = -\sin x + 1$ for $x \in [0°; 360°]$, showing all turning points and intercepts with the axes. (3)

5.5. Write down the equation of $k(x) = -2g(x) - 4$. (2)

[10]

QUESTION 6

In the diagram below, $AB \perp BD$. $CD = m$ units and $B\hat{D}C = \theta$ and $B\hat{C}D = \alpha$.



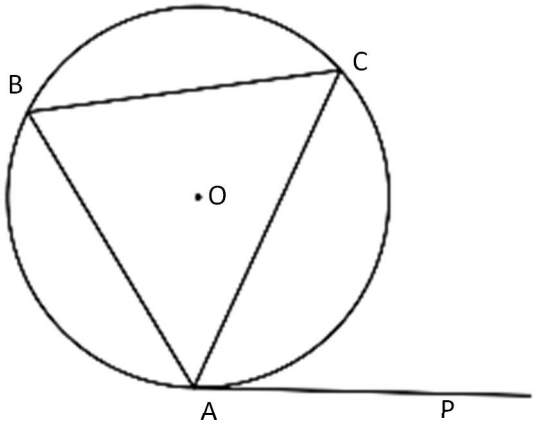
6.1. Write down the size of $D\hat{B}C$ in terms of θ and α. (1)

6.2. Prove that $AD = \frac{m.\cos x.\sin \alpha}{\sin(\theta+\alpha)}$. (5)

[6]

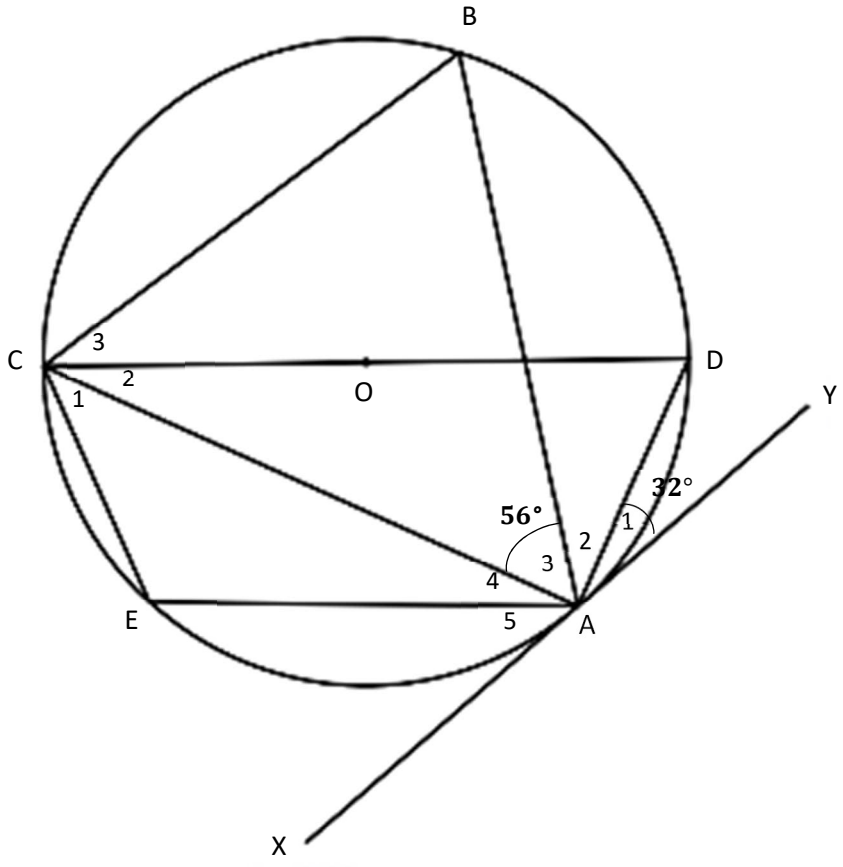
QUESTION 7

7.1. "The angle between a tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment."



Use the diagram above to prove the theorem that states that $P\hat{A}C = A\hat{B}C$. (5)

7.2. In the diagram below, XY is a tangent to the circle with centre O at A. E, D, C and B are points on the circle and AE, CE, BC, BA and AD are drawn. $\hat{A}_1 = 32°$ and $\hat{A}_3 = 56°$. DOC is the diameter of the circle.



7.2.1. Determine the sizes of the following:

a) $\hat{A}_2$ (2)

b) $\hat{C}_2$ (2)

c) $\hat{C}_3$ (2)

d) $\hat{E}$ (3)

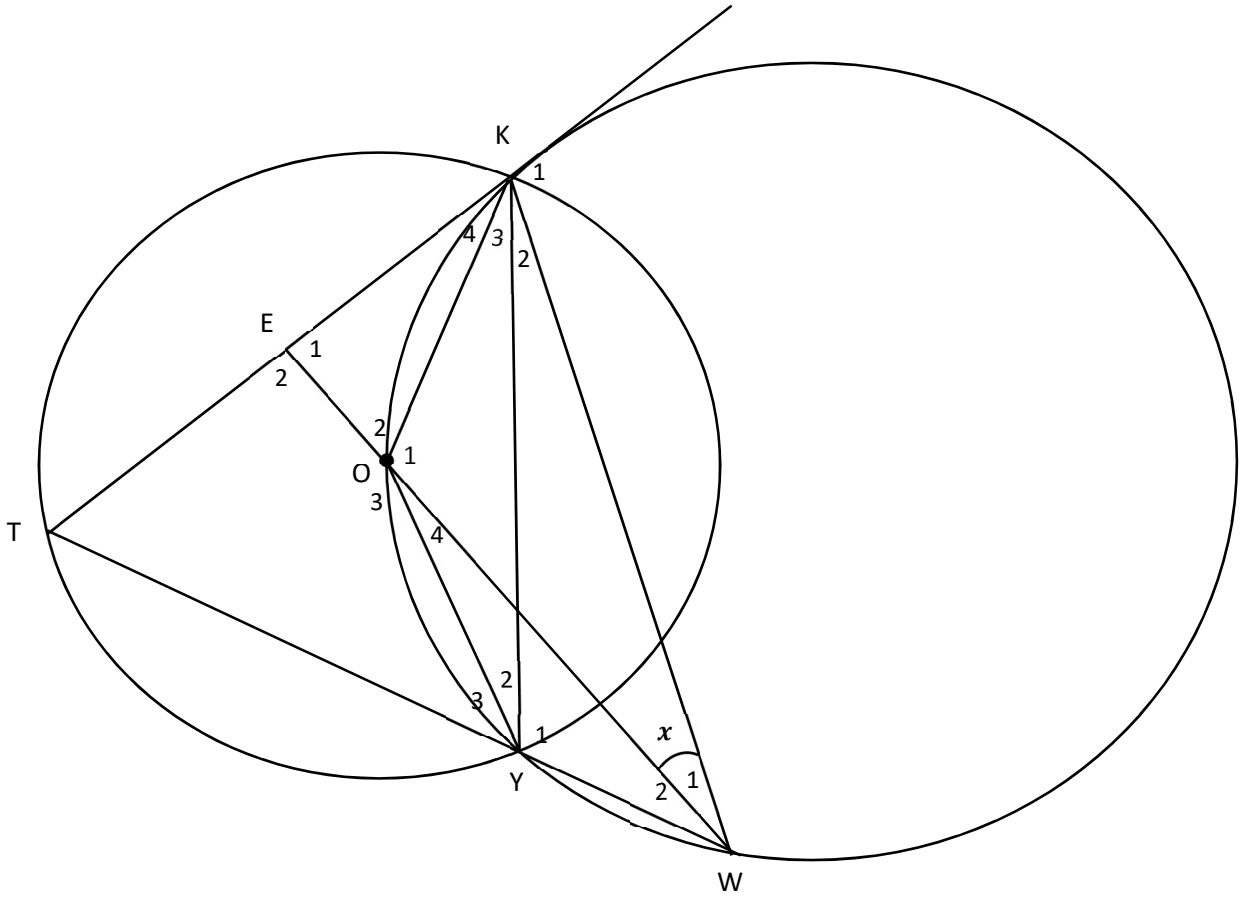
7.2.2. Is CD a tangent to circle CEA? Give a reason for your answer. (2)

[16]

EXTRA SPACE

QUESTION 8

In the diagram below, two circles intersect at K and Y. The larger circle passes through O, the centre of the smaller circle. T is a point on the smaller circle such that KT is a tangent to the larger circle. TY produced meets the larger circle at W. WO produced meets KT at E.



Let $\widehat{W}_1 = x$.

8.1. State, WITH REASONS, ***FOUR*** other angles also equal to x. (8)

8.2. Prove that $\hat{T} = 90° - x$. (3)

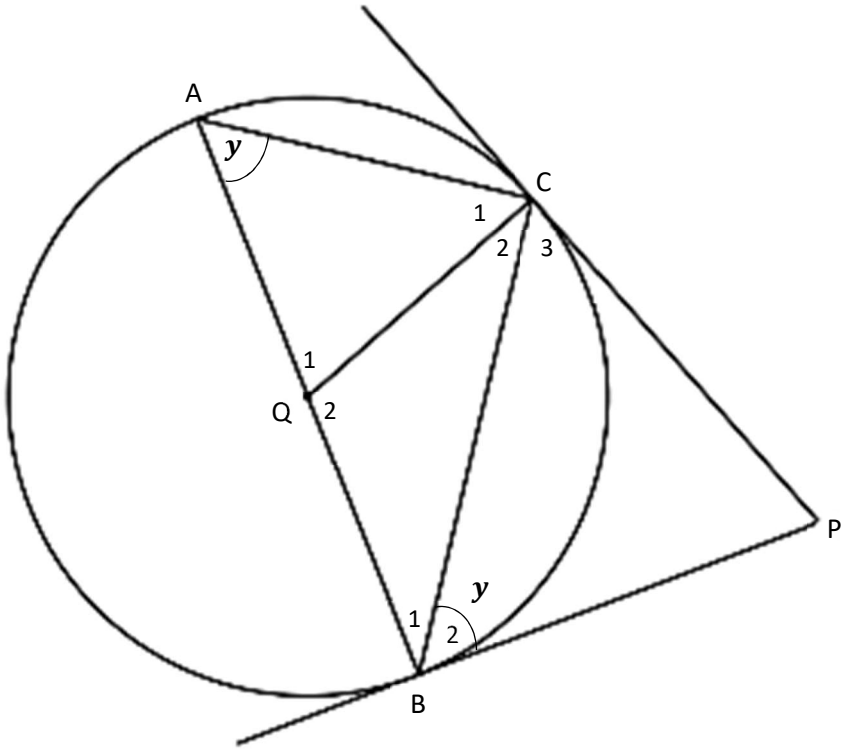
8.3. Hence prove that $KE = ET$. (2)

[13]

EXTRA SPACE

QUESTION 9

In the diagram below, BP and CP are tangents to the circle at B and C respectively. Q is the centre of the circle and AQB is the diameter of the circle.



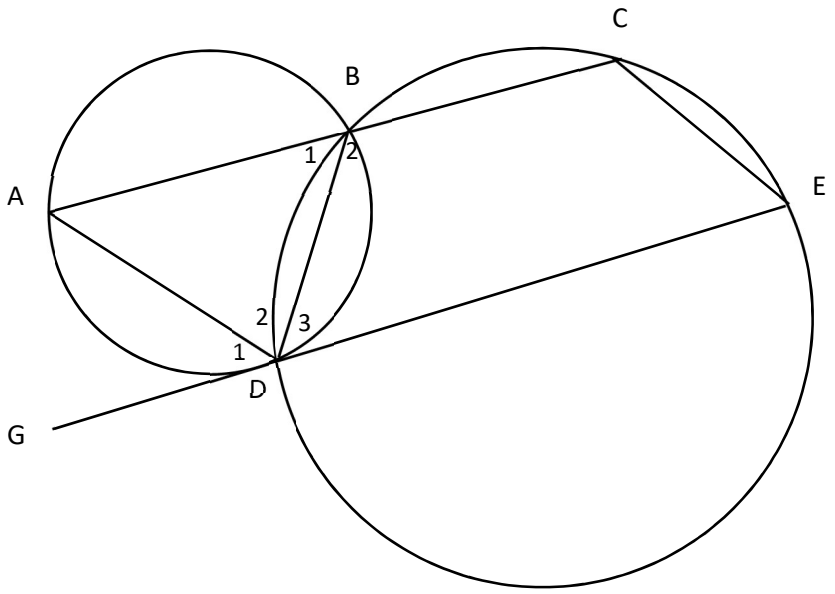
9.1. If $\hat{A} = \hat{B}_2 = y$, state ONE other angle equal to y. (1)

9.2. Prove that $QCPB$ is a cyclic quadrilateral. (3)

[4]

QUESTION 10

In the diagram below, A, B and D are coordinates on the smaller circle and B, C, E and D are points on the larger circle. GDE is a tangent to the smaller circle. $DB = AD$.



Prove that $ADEC$ is a parallelogram.

[6]

EXTRA SPACE

INFORMATION SHEET

FORMULA SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad A = P(1+ni) \quad A = P(1-ni) \quad A = P(1-i)^n \quad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad \mathrm{S}_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \;;\; r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;;\; -1 < r < 1$$

$$F = \frac{x\left[\left(1+i\right)^n - 1\right]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i} \qquad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right) \qquad y = mx + c$$

$$y - y_1 = m(x - x_1) \quad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta \qquad \left(x - a\right)^2 + \left(y - b\right)^2 = r^2$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A \qquad area\,\Delta ABC = \dfrac{1}{2}ab.\sin C$

$$\sin\left(\alpha + \beta\right) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin\left(\alpha - \beta\right) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos\left(\alpha + \beta\right) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos\left(\alpha - \beta\right) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\overline{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2}{n}$$

$$P(A) = \frac{n(A)}{n\left(S\right)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum\left(x - \overline{x}\right)(y - \overline{y})}{\sum(x - \overline{x})^2}$$